SOCIABILITY, DIVERSITY AND COMPATIBILITY IN DEVELOPING SYSTEMS: EVS APPROACH

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1. How to cook systems

1.1. EVS APPROACH

There is something about being a woman and thinking about cooking, even despite feminist protests. Suppose we would like "to cook a natural system", or at least to model it as realistically as possible. What tools do we need, and what should we do? Well, we need several elements, so let us take *P* elements and put them on the table. Are they a system? No, they don't do anything, and even if each of them would jump up and down on the table, they would still not be a system. We need them to relate to each other through their behavior. That is where the major differences exist between various models of multi-agent systems: their rules of relations, or interactions between agents.

There are thousands of examples of this in the modeling literature. Here we use our favorite - Ensembles with Variable Structure (EVS) [14, 17, 18], which are based on the following principles:

1. Non-locality of connections between agents.
2. Agents *randomly* check other agents with respect to compatibility.
3. *"Mutual agreement"* principle: connections between agents appear only when both agents "agree" to establish them, and if one agent wants to terminate a connection, then it breaks.
4. Population has a *diversity* of elements, defined via some parameters or vectors.
5. The number of connections to be checked/established is limited by the parameter of *sociability*.

In some EVS models we specified the diversity of these agents through the dynamics of a resource (such as limits on spending or receiving a resource, etc). In those models, each agent received and spent some resource at each time step, allowing the simulation of resource flow through the agent and through the system [15, 18].

As you see from the Table 1, our EVS models differ from other popular models by the non-locality of potential connections between agents (Principle 1), combined with systematic disconnections. Earlier models, such as those of random graph theory, percolation theory, cellular automata theory or self-organized criticality drew inspiration not only from physics but also from biology, which is a descriptive science of rather explicit phenomena. Biology, like classical mechanics, describes many local phenomena: interactions between objects that have immediate proximity and mechanisms based on such interactions. These local descriptions have received the greatest attention of scientists modeling natural systems. At the same time, there are a number of examples of non-local, indirect interactions in natural systems, which do not easily fit into those models. Locality, though, is a special case of non-locality.

Models	Connections	Disconnection rule
Random graph theory [9]	Local	No disconnection
Percolation models [7]	Local	No disconnection
Cellular automata [4]	Local and fixed	By a state of neighbors
Random boolean networks (the best review is [2])	Potentially non-local	No disconnection, but there are weights of a connection
Self-organized criticality [3]	Local	Random disconnection
Ensembles with variable structures [14-18]	Potentially non-local, limited by sociability	By compatibility or holding time

Table 1. Comparison of multi-agent models in terms of interaction rules.

Thus, we have a collection of interacting agents, where an interaction between two agents means a synchronization or inter-dependence in the behavioral dynamics of the two agents. When two agents demonstrate such an interdependence, they are said to establish a connection. While trying to establish a connection, an agent randomly checks a number S_j of other elements of the population, even while holding some pre-existing connections. We must now decide when to say that, “a system has emerged” as opposed to, “no, it is still just a set of agents peacefully carrying out exactly those simple steps that we asked them to”. Technically speaking, as soon as a pair of agents have “found each other”, they form a little system. On the other hand, we want something dramatic in our cooking, something big and significant rather than mere pairs of connected agents. We want to see something such as clustering behavior among agents, which was not prescribed in the rules of interaction that we gave the agents. So now a few words about clusters.

1.2. WHAT IS COMMON BETWEEN BENARD CELLS AND THE UNIVERSITY OF HAWAII?

Our rational upbringing suggests to us that we should hold on to something valuable as soon as we get it. It feeds our illusion that in reality there are connections that once established, forever: connections to our parents, friends, colleagues, the city that we grew up in, the organization that we are associated with. This illusion informs, for example, random graph models, or modern models of the Internet [5], in which connections are fixed, so we can count them. In reality, of course, nature does not sustain connections without using them. Consider first of two very simple questions: How to describe our workshop? What constitutes it? Would we have it if we were to place the same 23 people in the same room for six days without a requirement to exchange their knowledge? Probably not, as they should work “on science”. Would we have it if the location of the sessions were in some natural setting like the beach, rather than in the assigned conference room? Probably, yes. Does it depend upon our physical interactions, and on the exact choice of these very specialists? Probably, not. Would we have it if we were to do it on the Internet like some Internet Congresses? Probably yes. It seems that what is important here is having *interactions, scientific exchange*, while the physical arrangement of the workshop does not matter.

Figure 1. The dynamical structure of the University of Hawaii.

I hope you remember Benard's experiment with the heating of oil in a frypan, which at some point starts to produce nice geometrical and symmetric structures. What is common between these structures and the University of Hawaii, apart from the increasing heat outside? I would say that the common thing is *the* d*ynamics of structure,* a sort of virtual reality of connections between agents. These structures are dynamic ones in terms of the list of agents that constitute them.

You have parents, friends, and colleagues in your life as long as you keep meeting them and updating those interests that hold these connections together. You can still hold these connections long after their death, talking to them in your mind, or you can lose these connections after you stop sharing your interests with them, even if they are alive. When you move out of your city you should know that you cannot come back several years from now: it will be a different city, with different people and different priorities of places around. Nevertheless, this city will still exist, as well as some network of family and friends, just the "players" will be different. Relationships that is, interactions between agents constitute the structure of a natural system, be it the University of Hawaii, a government, a living cell or a Benard cell structure (Fig. 1).

In each given period of time these structures "update" the list of the agents which constitute it. Universities and governments regularly change their staff and associates, students and consultants. Cells update the chemicals that constitute their organelles on a constant and regular basis. In multicellular organisms, the environment of a single cell is more structured than the environment for a one-cell organism, but it is still an environment, providing building material for the cell, and changing "the list of actual elements" in the cell. We lose friends and family members from time to time, and we gain new ones, whether we like them or not.

Hopefully, that was a convincing explanation why EVS models have such interaction principles as number 2 and 3: random checking for a possible connection and disconnection in case one of the connected agents loses interest in the connection. These principles make EVS models very dynamic and make the clusters, i.e. the structures that appear within these simulations, very volatile. Other approaches that were mentioned above simulate equilibrium conditions, or, as in self-organized criticality models, certain special disequilibrium conditions.

1.3. PRESENTING DIVERSITY, COMPATIBILITY AND SOCIABILITY

What creates, replicates and develops these dynamical structures? What makes some agents part of a system and some – not? Scientists have searched for these factors within empirical settings and have identified many potential candidates. Here are the most popular candidates taken from the literature on complex systems:

- replicators - DNA, memes, and social rules keeping the structure of a system;
- phenotype – situation and history involving properties of an agent;
- environment – situation and history involving properties of environment;

Does this list exhaust the possibilities, or are there other global factors that could affect the emergence of a system and the clustering behavior of its agents? Jack Cohen likes to point out that 99.9% of frog eggs die after a normal reproduction cycle, so it seems that the first two factors are significant in 99.9% of cases. The success rate for any group of physical, chemical, social or economical agents to become a developed multi-element system is not any higher. Some environment takes over, whatever groups of elements remain, so we should focus on environmental factors. The environment expresses a much greater non-local impact than the other two factors. What are the global, universal environmental factors for the emergence of a system? Specifically, we are interested in the following questions: could such global factors of an environment as the diversity of a population, its sociability limits and limits of compatibility of elements have an impact on a system's development, and if so, what is this impact? First, we should say a few words about these factors.

Agents that might constitute a system could potentially, at some time, contact any other agent of similar nature. Still, physically there is always a limit on how many connections they could hold, or use, or check. This limit on immediate access to other members of the population, i.e. the number of connections to be checked/established per step is the parameter of *sociability* of an element in EVS modeling. For comparison, the concept of c*onnectivity*, which is popular in Internet modeling, refers to the number of actually extant links.

The other useful concept for studying interactions among agents is the *compatibility* concept, which was introduced in simulations of the interactions of diverse agents. These agents express their behavioral dynamics on the basis of their configurations or, if we use psychological terms, act based on their interests, goals and motivations. The compatibility concept is based on the fact that connections between agents in any natural system are a form of cooperation, or competition, oriented on the outcome of their activity, whether this activity is intentional or spontaneous. These synergy conditions occur in numerous examples in natural systems and are the focus of study of synergetics [8]. They were described at the cellular level within the theory of functional systems by Anochin [1], and appear more obviously at the level of individuals, groups, organizations and states.

To describe diversity and compatibility formally in modeling, we can take all possible traits, configurations, factors and characteristics of the agents and order up a vector space of these traits. We could imagine then the complete vector of interests for each agent, which characterizes the individuality of this agent within the space of these traits. If every agent has such "summarized" individuality, then we could formally compare agents using their vectors. This permits us to quantitatively define a difference in a configuration as a distance between "individuality vectors". We do not need to know the exact nature of each "interest", configuration or trait corresponding to some vector. We need only know the number of traits or dimensions, through which the differences between members of a group could be analyzed (dimension of the vector space of individual differences). This presentation of compatibility of configurations is easy to operate with mathematically in EVS-modeling. We have to note that compatibility of interests does not mean complete similarity of the agents; it is the result of the "synchronicity" of their configuration to have interaction and connection.

The relationships between these three formal environmental parameters of a population (sociability, diversity and compatibility) appear to have some subtleties which have an impact on the behavior of individual elements and contribute to the development of a system. Let's start with sociability.

2. Global games on the cooking table

2.1. SOCIABILITY TAKES OVER

Paul Erdos and Alfred Renyi started from a population of isolated nodes, randomly adding links between nodes, creating random pairs. During this addition of more and more links to randomly chosen nodes some pairs started to be connected with other pairs, and at some critical number of added links, the whole population of nodes became a big interconnected cluster. Thus, they found a first-order phase transition effect in clustering behavior. A similar effect was observed in various physical phenomena and was called percolation.

Our Compatibility model[1] [14] demonstrated a similar effect: with an increase of

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[1] The Compatibility model contained features reminiscent of spin glass models, considering an ensemble of N cells, each of which possesses a "resource of life" $\boldsymbol{R}$, and a $\boldsymbol{k}$-dimensional "vector of traits" $\mathbf{v}$, where each component can equal +/- 1. Each cell forms connections with other cells, and both the maximal number of connections per cell $\boldsymbol{S}$, termed the `sociability' and the rate of connection attempts $\boldsymbol{a}$ are fixed and identical for all cells. At every time step each cell $\boldsymbol{i}$ attempts $\boldsymbol{a}$ random connections, and its life resource $\boldsymbol{R_i}$ is adjusted according to the "quality" of its current connections. If $\mathbf{R_i}>0$, the cell dies, to be replaced by a new cell having the maximal life resource $\boldsymbol{R}$ and a random vector of traits. The quality of a contact between i-th and j-th cells is evaluated according to the traits of both cells:

$$q(i,j) = \sigma(T_{ij})(\mathrm{v}_i,\mathrm{v}_j) = \tanh(\gamma T_{ij})\sum_{m=1}^{k} v_{im}\cdot v_{jm} \qquad (1)$$

where (,) denotes the inner product of two vectors, $\boldsymbol{T_{ij}}$ is the duration of the contact between the cells $\boldsymbol{i}$ and $\boldsymbol{j}$, and $\boldsymbol{\sigma(T)}$ is the "efficiency of the contact" - for small $\boldsymbol{T}$ it linearly increases, and after several time steps saturates at $\sigma=1$. The quality varies between σk for aligned trait vectors, to $-\sigma k$ for anti-aligned trait vectors. For cell $\boldsymbol{i}$ having $\boldsymbol{n_i}$ connections and connection set $\{\boldsymbol{i_m}\}$,the value of the life resource at the next step is

$$S_i(t+1) = S_i(t) - \delta(n_i\{i_m\})$$

$$\delta(n_i,\{i_m\}) = \frac{\delta_0}{1+\alpha n_i} - \sum_{m=1}^{n} q(i,i_m), \qquad (2)$$

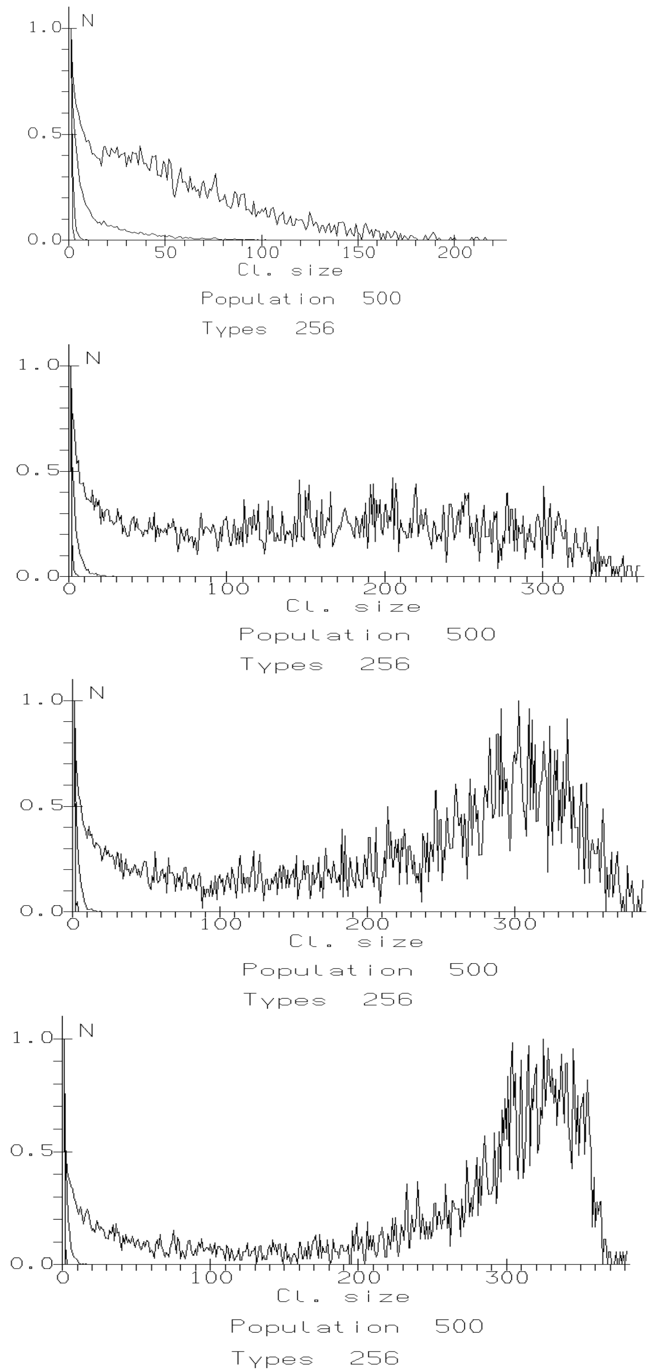
At each time step, a cell $\boldsymbol{i}$ is randomly chosen and a possible contact cell $\boldsymbol{j}$, which is not a member of the connection set of $\boldsymbol{i}$ is randomly selected. The possible profit of this connection for both $\boldsymbol{i}$ and $\boldsymbol{j}$ is determined by calculating $\delta(n_i,\{i_m\})$ and $\delta(n_i,\{j_k\})$. If the effect is beneficial to both $\boldsymbol{i}$ and $\boldsymbol{j}$ then the connection is formed.

agent's sociability (number of contacts that a given agent can check/hold per step), there was a critical value (critical sociability), above which a population created huge clusters of interconnected agents, and below which it represented a number of small groups clustered by interests. A first-order phase transition was observed as a function of sociability with the critical point $S_c=P^{0,6}$, where P is the population size. As we said, this S_c was a threshold between having a population organized into a large number of small clusters as opposed to a small number of large clusters (Fig.2). When sociability exceeded a critical value, it not only forced self-organization of a population into a few large clusters, it seemed to prohibit "small groups of common interests", as there were almost no small clusters left.

There are several comments that should be made about the role of sociability in such clustering. Our first point is something missed in random graphs theory: *the flexibility of the structure of connections in natural systems*. In graph theory links, once established, stayed without change. The rigidity of the links in random graph theory started partially to ease after the introduction of the so-called "small-world" architecture of networks. Thirty years ago, Granovetter proposed a concept of weak and strong links, and a clustering coefficient as a ratio between actually established and possible links [Grannovetter, 1973]. This meant that the links, once established, also stayed, but with various weights of importance, and it was implemented in network modeling. In contrast, in our EVS models links were far from being constant. In our EVS, the agents dropped the links for various reasons, simulating "traveling interactions" of particles or bodies in the real world. Even after the phase transition, i.e. after a population demonstrated the existence of a structure, relationships between elements in these large, "totalitarian", clusters of interconnected agents were generally dynamic, altering in time while still preserving the global functionality of the structure.

Second, it seems that *the value of maximal sociability existing within the population is more important than the distribution of sociability values within a population* (i.e. how many agents are with low, average or high sociability within it). The most interesting result in Erdos and Renyi's work was that the critical number of links to add for such a phase transition was equal to the number of nodes, i.e. on the average one link per node. It was difficult to judge what exactly was the range of sociability (maximal number of links, which a single node could have), as only the overall number of added links was counted. However, Erdos proved that the more a graph is growing, the more even is the distribution of links among the nodes, i.e. nodes are having approximately the same number of links. Does this mean that the critical sociability value (i.e. the number of holding links, that causes a large population to be unified into a large cluster) in the random graph theory would be $S_c=1$? No, it does not, as S_c is not the average number of links, but rather the maximum number of links that an agent could have in a population.

Investigations of actual clustering behavior in naturally occurring systems showed that a small number of elements that have sociability dramatically larger than the rest of the population could hold everybody together. "Networkers" called them "*hubs*". This ability to hold a large number of links makes hubs play the role of structuring elements within a population. The sociability of the Internet's hubs obviously exceeds critical sociability many times over, resulting in a clustering of the population into large clusters organized around these hubs.



Sociability = 20
Population size = 500
Diversity = 8 types
Activity per step = 4

Sociability = 50
Population size = 500
Diversity = 8 types
Activity per step = 4

Sociability = 70
Population size = 500
Diversity = 8 types
Activity per step = 4

Sociability = 90
Population size = 500
Diversity = 8 types
Activity per step = 4

Figure 2. Phase transition in clustering behavior.

This might be the case when there is only one agent within a population exceeding the critical sociability, while others might have a sociability only equal to one, and it would be sufficient to have a phase transition in clustering, holding everybody together. On the other hand, if the elements have on average five links, not one link, they might not be organized into a big cluster if none of them has sociability at or above the critical value for a phase transition.

What if a population does not have the "luxury" of having these "organizing" hubs? How does self-organization emerge in a population in which the maximal sociability value is not as dramatically high as, for example, the sociability of the Internet's hubs? Natural systems emerge before the development of such hubs. How do they do it? That is where a study of the critical values and the terms of a phase transition in clustering behavior become especially important.

The third point is that there is a *dominant role of sociability in clustering behavior over other formal parameters*. We investigated the roles of diversity, sociability and resource arrangements in clustering behavior in various EVS models [13-18]. Thus we used a wide range of diversity between agents, a wide range of sociability and various conditions of resource exchange, including "no resource parameter" conditions in our models. So far in these models, sociability values mainly determined whether a population of agents would be unified into some virtual system or would stay in small pieces. Sociability appeared to be the dominant environmental factor, determining the affiliative behavior of an individual element. Compatibility and diversity of the population had a much weaker impact on clustering, but at the same time influenced sociability and clustering behavior indirectly.

If sociability dominates in determining the phase transition in clustering behavior, why do we worry about the compatibility and diversity of agents in this clustering? We do so because in natural systems connections between elements of a structure are not only flexible and temporal but in many cases are not established at all because of an incompatibility of elements. That is why a real system eventually develops a hub, many times exceeding some formally sufficient number of connections: to hold together sub-clusters of diverse elements that would not be connected with the majority of the population unless they belonged to a chain of "small worlds" or "groups of common interests".

2.2. COMPATIBILITY MAKES A MOVE

Suppose that we have a population P of various agents, each having individual configurations, presented as a set of values given by D "traits"-components.

	trait 1,	*trait 2,*	*trait k,*	*trait D*
Agent 1:	$v_{11},$	$v_{12},$	$, \ldots v_{1k}, \ldots$	$v_{1D},$
Agent 2:	$v_{21},$	$v_{22},$	$, \ldots v_{2k}, \ldots$	$v_{2D},$
			………………	
Agent *j:*	$v_{j1},$	$v_{j2},$	$, \ldots v_{jk}, \ldots$	$v_{jD},$
			………………	
Agent *P:*	$v_{P1},$	$v_{P2},$	$, \ldots v_{Pk}, \ldots$	v_{PD}
Range of values:	$0\text{-}t_1$	$0\text{-}t_2$	$0\text{-}t_k,$	$0\text{-}t_D$

Thus, the first agent has an individuality expressed by components: $v_{11},\ldots, v_{1D}$, and the last agent has an individuality expressed by components: $v_{P1}, \ldots, v_{PD}$. For simplicity, let's imagine than we are able to use the same number D of traits in order to describe a configuration of agents.

These agents wander around in a neighbourhood of S_j agents, which is their individual limit of accessibility to others, randomly choosing S_j number of agents and "checking them out". We do not go into details of the specifics of what exactly should be synchronized between elements for the establishment of a connection, or what was not good enough when a connection was terminated or not established at all - we trust their judgments.

When their configuration is more compatible with some agent than with the one they currently keep a connection with, they switch to from a connection with the agent with maximal compatibility. In order to compare their configurations and find the optimal connection, EVS calculate the difference $\sigma_{ijk} = |v_{ik} - v_{jk}|$ between every two agents on each trait, summing these sigmas over the traits and finding for each agent j the minimal value of these overall sums of the differences:

$$\hat{\sigma}_j = \min_i \sum_{k=1}^{D} \sigma_{ijk}\ , \qquad k = 1,\ldots,D;\ \ i, j = 1,\ldots,P. \tag{1}$$

Our agents or elements have not only one, but S_j number of links to check/hold per step, which reflects their individual sociability. In this case, the agents deal with compatibility through a search minimizing their differences not just with one, but with S_j agents:

$$C_j\,(S_j) = \sum_{1}^{S_j} \sum_{k=1}^{D} \sigma_{ijk}\ , \tag{2}$$

where $C_j\,(S_j)$ is a compatibility value of *j-th* agent with its S_j links.

First, let us consider a "sterile" case which, although more attractive to computer scientists or mathematicians than to natural scientists, illustrates some of the subtleties of dealing with compatibility and diversity notions. If:

1) all agents in the population are different, $\min \sum_{1}^{D} \sigma_{ijk} > 0$, and

2) the range of values t_k of all traits has the same "step" u between values, i.e. $\min \sigma_{ijk} = u$, and

3) this range of values is the same for all traits, i.e. $t_1 = \ldots = t_k = \ldots = t_D = un$, and

4) the population represents all possible types and values of the traits, so

$P = (t_{max}/u)^D = n^D$, where t_{max} is the maximal value by a trait,

then the optimization of the structure of connections has a chance (after certain time steps) to find a stable solution. The compatibility value between an agent i and an agent j would be $\sum_{k=1}^{D} \sigma_{ijk}$, with a minimum value equal u. The number of combinations for a most compatible link with the minimum difference between two agents would be at the range of *D* to *2D*. In general, there are $n^d(n^d-1)$ compatibilities pairs to consider, as we exclude self links. The mean compatibility is given as:

$$\overline{\sigma} = \frac{\sum_{i=1}^{n^d}\sum_{j=1}^{n^d}\sum_{k=1}^{n^d}|i_k - j_k|}{n^D(n^D-1)} = \frac{D}{3}\left(\frac{(n-1)n^{D-1}}{n^D-1}\right)(n+1) \qquad (3)$$

For *D=1*, this mean equal *(n+1)/3*. In the case in which all traits behaved a fully independent, one would expect that mean would be *D(n+1)/3*. Clearly, in general, these traits do not act as if they are independent. For very large *D*, the mean tends to:

$$\overline{\sigma} = \frac{D}{3}(1-1/n)(n+1)\,, \qquad (4)$$

which is slightly less than the independent value. However, in the case of large *n*, the mean tends to *D(n+1)/3*. Thus the traits do tend to behave as if independent in the case in which they have a large range of values.

The dynamics of the links depends upon the actual compatibility between the agents forming the link, and the preferred links are those with the smallest compatibility values. The probability of holding a link will therefore depend upon its compatibility value. The range of compatibility values for a given agent *i* will depend, at least in this model, upon the specific choice of *i*. For any given agent *i* with sociability S_i, and any compatibility value m, one can calculate the probability that the minimum compatibility value over S_i, links is m according to the rather cumbersome formula:

$$prob(\hat{\sigma}_i = m) = \left[\frac{1}{n^D}\sum_{\substack{k_1+\ldots+k_D \ge m \\ k_j \le \max(n-i)(i-1)}} 2^{\gamma_n(i_1,\ldots,i_D;k_1,\ldots,k_D)}\right]^{S_i} \qquad (5)$$

We agreed first to consider a luxury case when in our population one can find an agent with any value of any trait. At every step, the agents would choose a smaller difference value and update the structure of their connections. Eventually, all agents in this situation would find the most compatible agents to associate with, making a nice difference-lattice of connected and optimally compatible agents, with the difference for each trait equal to just one unit *u* in *D*-dimensional space. It would be a "correctly defined problem with a final solution" for specialists on difference lattices. The system would find its optimum and would stay around it without much dynamics.

Unfortunately for mathematicians and fortunately for the rest of the world the first three conditions do not hold in real natural systems: the ranges of configuration parameters and the "steps" of these ranges are different, and often in a different order (i.e. the ranges t_i of all traits are different, and the units of the measurement of the traits

u_i are trait-specific), and the population has a limited number of combinations of types and values.

Let's look at what would happen in this situation to the structure of connections. In game theory, we would find many optimal solutions with even a small increase of diversity of options. A similar effect could be observed here. In eq. (3), the numerator would include a much wider range of possible values and so the average difference between agents would increase.

In general, each trait has its own unique range, so that the total number of possible configurations will be $\prod_{1}^{D} \frac{t_i}{u_i}$. The populations of elements in real systems do not represent all possible values of configurational traits, i.e. $P << \prod_{1}^{D} \frac{t_i}{u_i}$. With a limited number of types within the population the probability of getting a refusal for a connection (not finding a compatible type) increases, no matter how many steps the optimization of the connection structure runs. It means that agents often do not have a chance to find the "best-case scenario" and should oscillate between several "as good as it gets", trying to find a compatibility optimum.

What is good about this "mess"? This "everything is different" supports a dynamics of compatibility search, forcing elements to change/update their structure of connections more often and to not become frozen at a single optimum point or narrow region of values. It increases the degrees of freedom (number of combinations) in the structure of connections, complicating the compatibility search and making it more dynamic.

In this sense, a *uniqueness principle* (unique ranges of values for unique traits of unique configurations) makes natural systems more adaptive. It helps them to develop a region of optimal states to choose from under various conditions versus rigidity of the lattices with a narrow optimum around a fixed point of values.

2.3. SOCIABILITY CONFRONTS DIVERSITY

With even a slight increase in the range of traits, the structure of connections starts to become very volatile, but, as we know, natural systems demonstrate a great range on most of their configurational signs. How do they not decompose and manage to actually "play a system"? Here sociability is having a hidden "fight back" with the diversity. As you can see from equation (3), with the increase of sociability of an agent, the average difference between agents decreases, and so does their compatibility (eq. (2)).

It can be easily seen from both empirical observations and mathematical presentation that the higher the individual sociability of some agent, the higher its compatibility, as it is the sum of the differences with all the agents it is associating with.

With this optimization algorithm and from the law of large numbers a resulting compatibility value would oscillate around the expectancy values: $E_j = min\ Cj$ and the dispersion of differences between the j-th agent and his S_j number of contacting agents $B_j^2 = \Sigma(\Delta\sigma_{ijk})$. The central limit theorem would tell us that the probability of:

$\alpha B_j < Cj - E_j < \beta B_j$ has the limit with $S_j \rightarrow \infty$:

$$\frac{1}{\sqrt{2\pi}}\int_{\alpha}^{\beta} e^{-\sigma^2/2} d\sigma \qquad (4)$$

With an increase of sociability, the probability to optimize the structure of connections increases. The agents have a chance to choose "best from the best" in terms of their compatibility so that the value of C_j is approaching the expected minimum E_j and the range of σ becomes more localized. In such a way, sociability eases a compatibility search but also decreases the diversity of agents within the local clusters of interconnected agents. The simple selection of the most compatible agents would lead to a range of configurational values within a cluster that is much narrower than the range of such values within the population. But this selection rule is not only the way to decrease the diversity in a compatibility search.

In addition, at some point evolution provided systems with the ability to adapt their own configuration to their environment, an ability evolving from simple mechanical changes to complex adaptive intentional behavior. In natural systems once connected to beneficial elements, agents tend to demonstrate such adaptation behavior, developing traits that make them more compatible with their associates and decreasing traits that distinguish them from their groups. This can be observed in a wide range of systems but is especially visible in social, economic and psychological systems. Whether the cluster is a social group with its rules, an industry with its technical regulations or interaction between family members, all of these clusters tend to develop a common language, common infrastructure and common rules, making the contacts between the members of the cluster easier, but also making the members of the cluster more similar to each other. "*The small worlds*" architecture appears and develops with the development of such clustering, supporting "group norms" and "group language" within these small worlds of often connected agents (so-called "*strong links*"[6]) and increasing the isolation of a cluster from the rest of population with a resultant decrease in the cluster's internal diversity.

This tendency of "regression to the mean" does not succeed in the complete isolation and localization of the cluster's configurational values around some narrow range. The increase of sociability increases the cluster size of associated agents, and in a diverse population it means an increase in dispersion and thus an increase of the average differences, and again volatility in connections. At some point, agents end up being connected with so-called *"weak links"*, which are less compatible with their major configuration than the members of their small world, but it seems that in both real-life [6] and in our modeling it is something natural and beneficial. That is how diversity controls sociability and how sociability controls diversity.

2. 4. HOLDING TIME DOES NOT HOLD IT TOGETHER.

Although the inclusion of compatibility and diversity brings these models closer to reality, they are difficult to analyze formally. Even this simple complexity makes it difficult to fully appreciate the influence that sociability has over the clustering phenomenon. One day, Bill Sulis suggested a way to simplify the models even further and to "forget this diversity-compatibility mess". The compatibility structure was eliminated, leaving sociability behind. In its place was substituted a "holding time", a maximum duration over which an agent would hold a connection irrespective of the nature of the agent to which it was connected. The choice was eliminated from the model, all actions being random. At each time step, agents would eliminate connections held past their holding time, then randomly search for free agents to connect with. New connections would be formed up to the agent's individual sociability limit, but the search extended only up to this limit, so sometimes connections would remain unsatisfied. If anything, the model was expected to remain unsaturated. Disconnecting on the minimum of the two holding times ensured volatility in the connections. Short of making purely random disconnections, it seems difficult to think of a simpler system that still supports sociability in a non-trivial manner.

Even in such a simple model, one could observe an interesting and subtle phase transition in clustering behavior. At very low sociabilities, the system formed only isolated two-element clusters. At slightly larger sociabilities, clusters of all sizes could be observed to form. At slightly larger sociabilities, yet again mostly small clusters formed with occasional very large clusters. The spreading of the cluster distribution for intermediate sociabilities was reminiscent of a second-order phase transition, yet it occurred on a narrow range of sociabilities which is more typical of a first-order phase transition. Of course, this could be just a finite size effect. But in addition to this first transition, a second transition appeared. As the sociability was further increased, large clusters involving most of the system began to appear even more frequently. The cluster distribution was bimodal, one peak centred on the small clusters, one peak centred over very large clusters, with nothing in between. The maximum of the peaks shifted, as the sociability was increased, from being centred over the very small clusters to over the very large clusters.

Thus two phase transitions were observed. At low sociabilities, we went from a cluster distribution which was unimodal (peaked over small clusters) to uniform across the range, to bimodal (maximally peaked again over small clusters). As the sociability increased a second transition occurred, the distribution remaining bimodal, but shifting from a maximum over small clusters to one over very large clusters. Two order parameters were selected to reflect these two transitions. The first was defined as the size of the modal cluster divided by the size of the largest cluster. The second-order parameter was defined as the longest region of consecutive zeros in the cluster distribution.

The onset of these phase transitions varied as a function of the holding time. One result of this Holding Time model was a very important effect: the more "stickiness" a population has, i.e. the longer agents are stuck with certain connections because of their holding time obligations, the smaller clusters are within the population. A self-organizing system does not like when somebody forces its elements to be connected longer than it needs.

3. Stages of development of a system

The game between diversity, compatibility, sociability and size of the population seems to contribute to the development of natural systems. Let us briefly summarize a couple of possible scenarios of such development (Table 2).

As a prerequisite for a system to be developed on our cooking table, at a minimum two conditions should be met that are surprisingly similar to thermodynamic concepts. Actually, this is not surprising at all as phase transitions, and critical exponents were first described in detail in the thermodynamics literature [11]. In thermodynamics temperature, density and pressure are the parameters determining the changes of systems. With an increase of pressure (and so density) the access of elements to each other increases. In the Van der Waals' non-ideal gas model, an interaction term sums up all binary interactions between molecules and is proportional to the square of the density, i.e. the square on the number of particles in a given volume. It seems obvious that the requirement of the density condition, i.e. the distance between elements sufficient for interaction during the functional period, is universal in all natural systems.

Another control parameter for a phase transition in clustering behavior in thermodynamics is temperature, which is an increase in the number of contacts of an element with others per time unit. This parameter can find an analogy with our sociability parameter. We saw that with an increase in sociability, a phase transition in clustering occurs, and a system appears to be interconnected even while having very volatile connections and contacts. Examples of the emergence of systems arising from an increased rate of interactions, or an increase of potential access to other agents can be found in the dynamics of emergence and growth of a social group, of a city, or of business corporations.

3.1. STAGE ONE: ADAPTATION AND DIVERSITY INCREASE

Conditions of sociability and density provide elements with opportunities for more or less intense interactions. One problem is that up to this point there is no speciation of the population accumulated in these interactions, and so the range of configurational traits of interacting agents can be very broad, sort of "whoever comes we greet".

As mentioned above, agents in natural systems tend to adapt their configuration to the configurations of other agents that interact with them. This initial adaptation increases the "fractality" of configurational traits, diversifying their units u_i. In this sense, the diversity of interacting elements increases, thus reducing their success in holding a compatible connection. Wandering around each other, agents do establish temporary connections, which are unstable because of low compatibility and moderate sociability. It leads to the emergence of a conditional population of weakly connected (weakly synchronized) agents.

3.2. STAGE TWO: SOCIABILITY INCREASE AND POPULATION INCREASE

The diversity of agents and the instability of their clusters lead to a situation in which agents actively participate in "other worlds" or "other populations" interacting time to time with those agents. The usual strategy of a system against diversity is an increase in sociability.

If we use an analogy from thermodynamics, an increase of diversity among interacting agents leads to more frequent changes in contacts, and so to more intense interactions, which in thermodynamics is equal to an increase in temperature. This might affect the intensity of interactions between agents, which are not usually interacting frequently due to low density/far distances. In physical terms, an increase of temperature might give an agent sufficient energy to fly for a longer distance and to reach a more distant agent, and in social terms, an increase in the sociability of an agent might be associated with the bringing into a community its “relatives and friends” to participate in its functioning.

In this sense, an increase in sociability might lead to an increase of a population of agents that regularly interact. As we described above, an increase of sociability and population size allowed interacting agents to establish connections with more compatible agents. An improvement in compatibility leads to clustering behavior, and there is a chance for a population to switch from a number of small temporary groups to large clusters of interconnected “small worlds”.

3.3. OPTIMIZATION OF THE STRUCTURE OF CONNECTIONS

In section 2 of this article, we saw that an increase in compatibility, induced by an increase of sociability, helps to optimize the compatibility search and to find the local minimum of the difference function. This could be done under the condition of some “stickiness” in the population, allowing it to hold preferences and the most compatible associations.

This stage might be very short if there is little perturbation from the environment. We might call it “the spring” of the system, as it has the best adaptivity and the best potential. Its diversity and sociability are high, the population is growing, and the compatibility search makes the structure of connections more and more perfect. Unfortunately, this short period ends as the system starts to adapt to the fine structure of connections resulting from the compatibility search and starts to “cut” contacts that are not used for a while.

3.4. TOTALITARIAN CLUSTERING

Under stable environmental conditions, the adaptivity of some systems appears as a re-weighing of the connections, giving higher preferences to the most compatible agents. High sociability develops a tendency of “adaptation to the majority” in agents. It allows the system to make a “renormalization” of the configurational traits, decreasing the differences between connected agents.

The decrease of degrees of freedom (diversity of states), which are not used in the system within the most stable clusters, and high sociability create conditions for the development of uniformity within the whole population. If a change in an agent’s configuration is not stopped by environmental change, the population has a chance to develop stable clusters, structuring an increased rigidity of the system.

3.5. LIFE IN A MESS OR A PURE DEATH?

Table 2 shows two different scenarios for the next stage of development. The condition for this stage is simple, universal and unavoidable: the change in the environment. Under one scenario (A) we satisfy the system’s wish to have the highest compatibility between agents, with minimal differences between agents within the clusters and small differences between clusters. The population has high homogeneity, everything "flies with the same speed", and everybody "speaks the same language". Is this not a dream of any young manager, teacher or politician?

Suppose we have a perturbation of this system arising from its from the environment. This happens all the time in natural systems. Such a perturbation, i.e. an impact of some factors or elements causing a deviation from the established phase space, requires the system to respond with certain configurational traits, which in previous conditions were not required or were not popular.

Condition of development	*Stage of development*	*Popul. Size*	*Socia-bility*	*Diver-sity*	*Compa-tibility*
Density and sociability increase.	0. Wondering, gathering as separate elements or unstable small groups	Low	Higher	Low	Low
Diversity increase as adaptation	1. Establishing unstable connections with low compatibility.	Higher	Same	Higher	Low
Access to distant elements	2. Sociability (and so population) increase, compatibility increase and clustering	High	High	Higher	Higher within the population
Optimization of structure of connections	3. Finding and holding the most compatible elements.	High	High	Lower	High within the population
"Renormalization" and "structuring"	4. Decrease of degrees of diversity, which are not used in the system and stabilizing the most compatible connections.	Association of "small worlds" clusters	Lower	Same	High within the cluster, low within the population
Environmental request for a change, low diversity	5A. Decomposition and death due to involvement of elements in other systems.	Decomposition	Low	Lower	High within the population
Environmental request for a change, high diversity	5B. Remodeling of structure of connections	Ensemble architecture		Higher	High within the cluster, low within the population

Table 2. "Death of rigid" scenario (includes stage 5A) and "Survival of an ensemble" scenario (includes stage 5B).

Elements continue doing what they were doing. There are no miracles here: they continue calculating their differences and searching for the most compatible arrangements. A perturbation by definition causes a deviation in the configuration of some agents in some segment of the population. These deviant agents try to find compatible partners, but nobody prefers to establish a connection with them because of the resulting differences between their configuration and the rest of population. Thus, in totalitarian systems any ill agents as well as any novelty will die if the perturbation were is not very significant.

If we have a significant perturbation, it requires an adjustment of the structure of connections to the configurational change resulting from this perturbation. What does a system with limited diversity and strong ties between agents do in such a situation? It starts to decompose very quickly and dies. The elements can not longer find compatible members within this system due to the change in their configuration and start to participate in the clusters of the other systems.

Scenario 5B also has a population (association) of "small worlds" with the diversity within these world-groups probably lower than that within the population. The diversity of the population in this scenario still remains high. This means that the links between these "small worlds" are weak, and these world-groups are not super-stable. Also, from time to time, their members could establish connections with some average-compatible agent, and a member of another small world could temporarily occupy the vacant space. This is an "*ensemble*" architecture of the population, when diversity is not only allowed but is required both between the clusters and within the clusters.

These two scenarios again show the conditional benefits of unification and diversity. Unification, which happens under high sociability conditions and regression to means is beneficial for the functioning of a system under unperturbed conditions. The diversity of the connections, and imperfections in compatibility, create the set of just "good enough" connections, producing an ensemble of diverse elements, ready to establish connections with even more diverse elements. The diversity of an ensemble is its lifesaver, as it makes it more adaptive to a change of environment. The most stable, rigid and non-adaptive natural systems are those, which have little diversity of elements, and the most unstable, but the most adaptive systems have a high diversity of agents.

4. Conclusions

We discussed that:

- Sociability is the major factor affecting clustering behaviour in a diverse population;
- Diversity and compatibility have ways to control sociability, and sociability has ways to control the diversity;
- Diversity, compatibility and sociability could be considered as global factors affecting the development of a system, as its interaction within the developmental stages defines the specific of these stages;
- Diversity of agents and an ensemble architecture of connections are beneficial for the survival of a natural system functioning in a changing environment, while unification is beneficial in stable conditions;
- Establishment of interactions between agents of a population on the basis of

compatibility of their configurations is associated with a first-order phase transition (in clustering behaviour), common in physical systems;

- The stickiness of agents decreases the possibility of a 1st order phase transition but leads to a second-order phase transition, common for biological systems.
- Compatibility of interests in making a connection makes a phase transition from a population of small clusters to an all-unified population smooth. Absence of compatibility makes this transition sharp.
- The artificial holding of a connection instead of compatibility condition delays the phase transition in size of population and sociability conditions but then makes the phase transition very sharp.

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