Chapter 4.

Modeling in Life Sciences
Principles, Concepts and Phenomena of Ensembles with Variable Structure (EVS)

Irina Trofimova,  
Laboratory of Collective Intelligence,  
McMaster University

Abstract. Four concepts in the modeling of the behavior of a multi-agent system are discussed: diversity, «compatibility» of agents, sociability, and the flow of resource. These concepts and their corresponding parameters were used in a set of models termed «Ensembles with Variable Structure» (EVS). EVS-approach represents interacting groups (populations) with a flexible structure of connections and a diversity of elements (agents), where agents possess an abstract set of characteristics and seek to form connections with other agents according to the degree of compatibility between these characteristics. EVS models show some system phenomena and formal relations between diversity, sociability and size of a population.

1. Principles of Ensembles with Variable Structure (EVS)

Adaptation of a scientific language to the real topology and dynamics of the world requires a study of “invariant group solutions” present among natural multi-agent systems. Such solutions could include the relationships between the universal characteristics of any population: size, diversity, degrees of freedom for interaction and density of agents. Knowledge about the relationships among these formal characteristics could help in the development of new tools for the measurement of dynamical systems.

Consideration of multi-element systems, be they brain or body of a subject, groups of subjects or organizations, leads to the necessity of formal analysis of interactions between the elements. Random graph theory, percolation models, interacting particle systems, spin glasses, cellular automata, and random boolean networks all constitute models of interacting multi-agent populations. Most models, however, consider formal populations with identical elements or possessing only a small diversity of types, strategies or rules. Also, the agents of some of these models interact only locally (cellular automata, networks), or the connections, once established, are fixed, as are the vertices (percolation model, random graph model). This simulates equilibrium conditions.

A set of models that we developed in collaboration with Alexey Potapov and Nikolay Mitin from Keldysh Institute of Applied Mathematics (Russia) and with Dr. William Sulis from McMaster University (Canada) are called Ensembles with Variable Structures (EVS). In this EVS modeling approach, we are trying to follow the principles of functioning of natural systems, rather than to copy their structures.

Briefly, the principles of the EVS approach are:

- Simulation of multi-agent parallel distributed systems.
- Non-locality of connections between agents.
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- The structure of connections between elements is very dynamic and stochastic.
- The Population has a diversity of elements, defined through some parameters or vectors.
- Agents randomly check other agents on the matter of compatibility, based on their diversity.
- The number of connections to be checked/established is limited by the parameter of sociability.
- Each agent receives and spends some resource on each time step, that allows it to simulate resource flow through the agent and through the system.
- In some models agents can change their characteristics to get more profit (adaptive algorithm);
- To achieve these properties, the EVS approach used a Monte-Carlo method for a search of connections and a spin glass style simulation, ordering the diversity of elements by an expanding dimensionality of spin characteristics.

2. Concepts of EVS

2.1. Diversity phenomena

Physicists and mathematicians are used to dealing with the diversity of an object using statistics: we have some configurations that are more common, and we construct parameters of measurement for these configurations. Other objects, having greater variety and less frequency of appearance, are treated as just noise, that annoys mathematicians, forcing them to add special «noise» terms to their otherwise perfectly compact equations. To some degree, they are right: in the language of probability theory, there are more likely events (properties) and more rare ones.

The most probable properties constitute the signs of an object and allow us to classify this object somehow. In this sense «spikes» in a probability distribution give us existing types of existing objects, and low regions in the probability distribution are perceived as deviations from these types, as noise.

Let us have a look at some qualitative diversity characteristics, which are not presented in mathematics and physics. The first and most obvious is the fact that no natural system is composed of identical elements. None. If we are talking about a natural system, we are talking about the union (whether it is structural or dynamical or functional union) of different elements.

The second is that the difference between elements composing one system has certain limits: a naturally occurring system cannot be an association of just any number or kind of elements. The emergence of such a system happens only given a particular diversity of elements, and this diversity is not arbitrary.

Third, the number of elements of each type is different for different types of system: it seems that the diversity of systems depends upon the distribution of diversity between the system's elements. For example, a hierarchical system (such as group of ants or organization of people) has a subordination structure, where there is very small percentage of elements in the «leader» (or control) position (absolute number usually comes to 1-2 elements), a greater number of «sub-leaders» and an impressive percentage of «corresponding», «working», or «serving» elements. Heterohierarchical systems (for example, chemical structures or markets) do not have such a separated control device, or an element, ordering the behavior of a system, but has a larger group of elements that provide the coordination between all elements of the system. Later we'll argue that optimally a system should have both types of distributions.

Developing the image of «how many of what» a natural system is supposed to have, we can find that usually, the content of visible elements of a system is not a complete list of a system's elements. Natural systems are always open and dissipative dynamical systems: traditionally we describe in this case how economic organizations or animal bodies have a constant resource exchange with their environment, but the same is true for chemical and physical systems. In real nature chemical structures have active «internal» and «external» life: their atoms vibrate and sooner or later go out of even very stable structures, and other atoms
could be associated, or the whole structure interacts with other structures. On the level of elementary particles, once appearing they don't stay until the meeting with other particles. Virtual particles can be created and disappear any time, and under special conditions, these virtual particles become the real ones.

All this we say to use the concept of «grass» type elements: their numerous presence surround the main structures (as herbivores should have enough plants around, the economy has money flows, proteins - more simple chemical elements, pion clouds surround neutrons and protons, and muon clouds surround electrons). These «grass» elements serve the main structure not only as a food but as «garbage bags» as well. They give the possibility to these systems to decompose certain elements when it is necessary. The percentage of such «grass» elements is also not arbitrary: we could recall the «prey-predator» models to understand it. Too small an amount of «grass» will lead to a reduction in the number of elements that use it, and too much grass will increase this number for a while, after which an increasing number of predators will delete extra «grass and garbage bags»: in both cases, the percentage of «grass» will come to some limits, within which the relation between feeding system and «grass» will oscillate.

Finally, the most significant aspect of the relation «object - grass environment» from the statistical point of view is that the number of these «grass» elements, details of natural «Lego» that might constitute an object is much-much greater than the number of our natural objects under study. It leads to the not just «noisy» environment in the description of the behavior of an object. As all these «grass elements» have their own life, all their activity together leads to such a mess, that any deterministic description of the behavior of an object sinks in this ocean of «small influences and reasons». The best approach proposed for the presentation of this situation is the stochastic approach.

The fourth aspect of diversity that we would like to mention here (but not the last generally speaking) is the functional differentiation of elements and poly-functional use of the elements. An element will happen to carry out one function or another based upon the ranking of its ability relative to the other elements, and it may change its function should it receive a better rank for another ability, or should some other element appear with a better ranking than its own in this specialization.

These features demonstrate that ignorance of the diversity of elements in our modeling of living systems might lead to our missing out on some basic properties of these systems.

2.2. Compatibility concept

The compatibility concept was introduced in simulations of the interactions of diverse agents, acting from the point of view of their own interests, goals and motivation. This concept is based on the fact that connections between elements of any natural system are not established unless there are reinforced by the motivations for the similar outcomes of their activity, and this outcome becomes the subject of cooperation between these elements. This was observed in psychophysiology within the theory of functional systems by Anochin [1], and this compatibility principle is more obvious at the level of individuals, groups, organizations and states' interaction.

In order to deal with it on the formal level, we can take all possible motivational factors and characteristics («interests» of the agents) and order up a vector space of these interests. We could imagine then the complete vector, which characterizes a certain agent within the space of these interests. Such interests need to be interpreted broadly, as motivation to certain action in economical, physical, psychological, social, aesthetic, intellectual, or informational sense.

This compatibility of interests is easy to operate with mathematically in modeling: every agent has a «summarized» individuality and «motivation», and so we could formally compare them using their vectors. It permits us to define a distance between every two «individuals» quantitatively. We do not even need to know the exact nature of each «interest», or trait, which corresponds to some vector. We need to know only the number of traits, in which space the differences between members of a group could be analyzed (dimension of the
vector space of individual differences). In this summarized presentation agents might receive some progress (positive compatibility) or distraction (negative compatibility) from the interaction. We applied this concept of compatibility in our Compatibility model [11], [12], [15], [16].

We have to note that compatibility of interests does not mean complete similarity of the agents; it is the result of the «synchronicity» of their motivation, which improves the effectiveness of their interaction. We also explored the other presentation of diversity and compatibility, which is connected with the resource exchange (R-compatibility). R-compatibility or compatibility through the direction of resources appears when an agent, intending to spend a resource contact an agent wishing to receive it, and verse versa.

2.3. Flow of resource

The concept of resource and its use in the modeling of organizational activity opens up possibilities to both define the functional diversity of elements inside natural systems and to analyze the dynamics of their interactions. We consider the concept of resources broadly: it could refer to energy, matter, chemical elements, time, information, money, service, emotional exchange, and so on.

EVS uses this concept in order to simulate a principle of openness of natural systems and the dissipation of energy or resources. In the majority of EVS models, each agent receives some resource and spends some resource at each step of time. Individual differences in the limit on input and output resource during the resource exchange between agents might play a role in functional differentiation between agents. This role was studied in our first Functional Differentiation model (FD-1) [13]. This model showed a phenomenon, which is intuitively well-known: under the condition of variable structure of connections and exchange of resource an amount of resource received from the other agents is approximately the same for various agents. Still, the strategy of spending a resource plays the biggest role in functional differentiation. The majority of elements of a natural population are usually exposed to incoming resources and possibilities more or less equally, and the differences between these elements lie mainly in operating with these resources and possibilities.

If agents usually receive a resource with the same probability, but spend it with various strategies and distributions, it is important to know what «spending» parameter of R-compatibility to use in modeling. In our Resource model [14], we analyzed three such «spending» parameters:

- fixed necessary expenses per step (life expenses), which an agent cannot avoid;
- maximum of expenses per step (including the cost to have a connection);
- maximally allowed percentage taken from the existing resource, which an agent can spend.

The results show that even when agents could change their «individuality», i.e. values by these parameters, only one «spending» parameter plays the major role in self-organization of a population. This parameter is the maximally allowed percentage taken from the existing resource, which an agent can spend. In our next Functional Differentiation model (FD-2), we used this fact and ordered the individual differences of agents based on the threshold of expenses. This threshold is a per cent of the belonging to an agent resource: an agent can spend a resource if it exceeds this threshold and is motivated to receive a resource if it has less than a threshold in his possessions.

2.4. Locality versus non-locality, or the concept of Sociability

Probably the assertion of the non-locality of connections between elements inside the animal world, psychological, social and economic systems is obvious even for a child, and, of course for people inside any organization. People interact with one or another member of the environment; groups interact with groups as entities or single people, and flies interact with all
sort of animals. Less obvious non-local interactions occur within cellular communities. The lack of appreciation of this led many mathematicians to the principle of locality of connections between the elements in their models. This locality principle is used in most popular multi-agent models, especially cellular automata (from [18]. According to the principle of locality, each agent in these models has already established connections with its neighbours, the connections are fixed, and the number of such neighbours is very limited [2-5, 9-10]. This means that the structure of a system is defined by its connectivity and does not change. Such a structure makes all agents to be active in the input stage.

It is true that in natural multicellular systems, such as bodies of brains, there is local cell connectivity. Real neurons however 1) are silent most of the time or make random spikes from time to time, and thus do not really participate in the network; 2) each neuron of the adult person has thousands of dendrite connections with other cells and sends axon branches to thousands more, many of these at considerable distances - it means that each neuron swims in an ocean of connections, an enormous space of possibilities for contacts; 3) the connections between them are not equal as a consequence of the neuron's communication vehicle (it is not only a question of excitatory or inhibitory types). In point of fact, connections between neurons possess a large variability that allows our brain to code information not only by the morphological structure of connections but by dynamical patterns in time; 4) neurons can communicate through neurohumoral factors with other neurons and with other organs. The same is true for somatic cells: connection and regulation between them are not via physical closeness, but through the chemical exchange using different kinds of fluid matter that flows through it. If one cell has a special state that can influence another, «long-distance» cell (to help, to break, to stimulate, to inhibit, to feed, etc.), it is just a question of time (sometimes just milliseconds) for this cell to establish this influence.

Questions of locality vs. non-locality in natural interactions are not new, physicists have discussed these for centuries. The main result of this discussion is that there are both local and non-local interactions in natural systems, which reflect the stability of structures and the flexibility of development. There are at least three states of matter - solid, liquid and gas, originally introduced in statistical physics to distinguish between the highly ordered state of an ice crystal say, flexible connections between atoms in liquid water and the highly disordered state of gas. In the case of a solid, local neighbourhoods determine the «life» of an element, in the case of gas the state of all elements (atoms) determines the state of an element. The liquid state has a special ability to have both properties: hold a structure, and change it under necessary conditions. This special ability gave to our liquid-based life systems an advantage over ancient rocks and gases of the Universe to produce high complexity and self-regulation.

From this point of view, we believe that the principle of flexibility and variability of connections in our EVS approach is more adequate for the modeling of activity of natural systems (organisms, groups, organizations or other communities) than traditional neural networks or cellular automata approaches. According to this principle, an agent could potentially establish communication or other joint activity with any other agent of the population or group. However, they cannot establish such connections with everybody simultaneously - we could potentially make contact with any person in the world, but not simultaneously with everybody. Thus we define the concept of sociability as the maximum number of contacts that an agent could hold at any step of the time. Generally, sociability is the characteristic of an agent which is describing the structure of its connections inside a population (both limit and the distribution of connections).

3. Summary of models

Table 1 presents a summary of five models developed within the EVS approach: Compatibility model, [12, 16], which had an earlier name as Collaboration model ([11], [15], Adaptation model [14], [15], Functional Differentiation (FD-1 and FD-2) models [13], [17], and the Resource model [14].
As was said before, the EVS approach used Monte-Carlo and spin glasses simulation, ordering the diversity of elements by an expanding dimensionality of spin characteristics. For example, usually spin has 2 types of states, but in two dimensions we can receive 4 types, in 3 dimensions we can receive 8 types, and so on. Agents in populations possessed an abstract set of characteristics, seeking to form connections with other agents according to the degree of compatibility between these characteristics. Each connection carries with it a relative valuation on the part of the agent forming it, and the agents attempt to optimize their valuations over time. We considered the situation in which the distribution of connections is uniform throughout the population: every element can potentially establish contact with every other agent with equal probability, and hold this contact if it is profitable.

Population size for models under study were 20, 100, 200, 300, 400, 500, 1000, 2000. All runs took 5000 steps (for smaller populations) or 10,000 steps (for populations 500, 1000 and 2000).

The agents of the Compatibility model interacted on the basis of their individual vectors of interests, and these vectors were ordered in various dimensions to provide a diversity of interactive spines. In this model, individual agents attempted to minimize the costs associated with the establishment of cooperative links with neighbouring agents. These costs varied according to the ‘compatibility’ between agents. The links were dynamic, changing with fluctuations in costs. Population size, diversity, sociability and contact rate were tunable parameters. The object of the study was the formation of connected components, and the behavior of affiliation was considered (formal dynamics in cluster formation under various values of diversity and sociability of agents). To obtain a closer look at the dynamics, we examined the following: number of contacts established per time step, number of agents dying per time step, the average age of agents per time step, number of the cluster formed per time step, and the local order parameter defined as mean cluster size/maximal cluster size per time step.

The individual differences of agents in both the Adaptation and Resource models were not abstract traits, but the characteristics of output resource (limit of it and a percentage of it derived from the residual of an agent). An agent could change its configuration and become closer to the «average individuality». Each agent attempted to minimize its costs depending upon the degree of similarity in type between itself and those agents with whom it had forged links. Thus the compatibility of agents here was based on their similarity of spending limits: the more similar their individual spending strategies, the more resource they receive. The initial distribution of values by traits was random, as was the formation of links.

The Resource model compared this criterion of optimization with another, more «economical» criterion, when an agent does not receive special profit for similarity, and just should store as much resource, as it could. In addition, the Resource model considered two types of limitation on a number of connections: the overall limit of allowed connections and individual sociability of each agent.

The agents of the FD models based their compatibility on the direction of the resource. While in our original Functional Differentiation model (FD-1) we did attempt to order several limits on incoming and outcome resource for each agent, the FD-2 model uses one «combined» criterion for R-compatibility, related to both income and outcome or resource. This criterion is an individually assigned threshold on spending a resource: below it, an agent tries to receive a resource and seeks a compatible agent who wishes to spend some resource; when this threshold is exceeded, an agent searches for a connection with another direction of resource, i.e. with an agent who wants to receive. The value of this threshold varied among agents and was ordered as a percentage from the holding of the resource by an agent.

We would like to emphasis that in the EVS models, connected components proved to be highly volatile, with large fluctuations in connection structure occurring for all parameter values - such as the real dynamics of everyday life connections between people or organizations. This corresponds somewhat to the concept in the complexity theory of structural stability.
Table 1. Summary on EVS models

<table>
<thead>
<tr>
<th>EVS Models</th>
<th>Diversity (N of types)</th>
<th>Sociability</th>
<th>Compatibility</th>
<th>Criteria of optimization</th>
<th>Phenomena</th>
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<tbody>
<tr>
<td>Adaptation</td>
<td>30x20</td>
<td>Limited equally for all agents, in each case in a range 10-80% of population size</td>
<td>By spending resource parameters</td>
<td>Similarity with others by spending limits</td>
<td>In spite of allowed self-change of agents, there are small oscillating clusters of agents which are different from the majority of a population.</td>
</tr>
<tr>
<td>Resource</td>
<td>30x20</td>
<td>2 cases: 1) limited overall a system; 2) limited individually, in a range 10-80% of population size</td>
<td>By spending resource parameters</td>
<td>2 cases: 1) maximization of a holding resource, 2) similarity with others by spending limits</td>
<td>Among the spending parameters, the most important for self-organization is a % of the remaining amount which is allowed to spend. A limit on connections (Sociability) should not be applied on a system, only on agent's activity.</td>
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<tr>
<td>Compatibility</td>
<td>4, 8, 16, 64, 256</td>
<td>Limited equally for all agents, in each case in a range in a range 10-80% of the population size</td>
<td>By a vector of interests</td>
<td>Similarity of interests</td>
<td>1. The formation of large clusters appears to be a function of the sociability and an inverse function of population size. A 1st order phase transition appears at $S_c=p^{0.6}$, where $P$ is the population size. Large cities effect, Small towns effect. Optimal group size effect. Increase of diversity decreases a size of clusters and vice versa.</td>
</tr>
<tr>
<td>FD-1</td>
<td>64, related to the limit on incoming and outcoming resource</td>
<td>5-70 % of the population size</td>
<td>By the direction of resource (R-compatibility)</td>
<td>Exchange of resource in order to survive</td>
<td>1. Resource income and outcome individual limits of agents and their individual sociability determine their functional role in a population. 2. Parameters related to spending a resource play more important role in functional differentiation, than limits of incoming resource.</td>
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<tr>
<td>FD-2</td>
<td>Equal to N of agents and related only to the threshold between the need to spend and the need to receive a resource</td>
<td>5-70 % of the population size</td>
<td>By the direction of resource (R-compatibility)</td>
<td>Exchange of resource in order to survive</td>
<td>The ratio of functional types stayed within the limits: 18-25 % of «spenders» to 75-82% of «holders-receivers», under the condition of highly diverse population. A decrease of diversity of thresholds and a decrease of sociability leads to a decrease of % of «spenders». Sociability determines the asymptotic dynamics and variability of clusters and connections.</td>
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This concept means not the stability of visible structures and their sizes, but a certain invariance of functions that describe the states of these structures of spatial or temporary deformations. Thus the system has a flexible, varying structure, but keeps the basic attributes, in spite of the fact that the constructing structure elements were replaced. The nonlinearity of complex systems determines multi-alternativity of display of their properties: the element can critically change the structure of connections, and input/output resources flow through it because of the change of the rank in population by some characteristic, although its formal parameters remain constant. Its rank in population can vary with the change of environment; thus, the system will put it in another place, and another element will occupy its previous place. Such a system approach describes the role of an element of the system that depends upon the element's place in the structure.

4. Some Phenomena

4.1. «Big cities effect»

Reasons for our social behavior are addressed usually to a variety of factors: genes, personality traits, developed during ontogenesis, the social situation in the reference group, political and economic situation on a large scale. The study of the «Compatibility» model showed how «system», social factors might have an impact on affiliative behavior, i.e. the phase transition between many small separate groups, and the union of agents into a small number of big clusters, which we can interpret as the appearance of a system.

In order to study more easily this change, it is usual to introduce a new parameter, termed an order parameter. An order parameter is a parameter that serves to distinguish between distinctive ordered states of a physical system. For the purposes of our study, a convenient order parameter is the ratio of the mode of the distribution to the maximum obtained cluster size. Clearly, this order parameter will be small when the distribution is highly skewed towards small clusters, and large when it is skewed towards big clusters. This does break down when the maximal cluster size is also small, but this effect occurs only for very small maximal connection values and is not significant here.

The first social effect of the Compatibility model that we would like to describe was found in the large populations (such as 2000), so we called it an «effect of big cities». Theoretically, one would expect that the distribution of cluster sizes would be a monotone decreasing function of cluster size. Moreover, such a distribution should follow a power law [6]. The model demonstrated such behavior for large population sizes. For large populations, the expected behavior was followed regardless of the diversity of traits (type), activity or contacts.

The dynamics of these connections appeared in the formation of clusters of interconnected agents which appeared and disappeared over the course of the simulations. The affiliation of the group (defined as the ratio of the mode of the distribution of cluster sizes to the maximum generated cluster size) is close to 0 when the distribution is highly skewed towards small clusters (as on the Fig.1), and close to 1 when it is skewed towards big clusters. As you can see in the figure, there are many small clusters (groups of affiliation, group by interests, some unions that include individuals more often connected with each other then with anybody else), and very few if any big clusters. We can understand this situation in terms of individual differences also: if we want to find any characteristics that unify large groups of people, then we'll find only a few of them: for example, sex is probably the most significant sign for the division of people into groups (a group of men in one cluster, and a group of women - the other); age groups could give us big clusters too. However, the more characteristics that we can imagine, the more groups we could find: a group of people that likes milk, a group that does not like John Travolta, a group of people having only three children with a son as youngest, a group of people who own a large enterprise, or of people who listened to the afternoon news last
Monday about the situation in local educational funding. Such behavior is theoretically understandable and expected for every system.

<table>
<thead>
<tr>
<th>Sociability</th>
<th>Cluster Distribution Functions</th>
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<td>5</td>
<td><img src="image" alt="Cluster distribution for Sociability 5" /></td>
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<tr>
<td>30</td>
<td><img src="image" alt="Cluster distribution for Sociability 30" /></td>
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<tr>
<td>90</td>
<td><img src="image" alt="Cluster distribution for Sociability 90" /></td>
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</table>

**Figure 1.** Cluster distribution functions for population 2000. The x-axis represents the size of clusters, the y-axis represents the number of such clusters normalized against the mode.

There is another side of an «effect of big cities». If we decrease the possibility of an agent to be in contact with others (sociability) in such populations, we see a tendency towards a transition in the other direction: the smaller sociability, the bigger clusters dominate (Fig.1, a). This situation probably means that a deficit of contacts for the large populations could create the conditions for the less adequate choices that an agent could make and so for the more stable affiliation in some large groups. Such a situation leads again to possible ideological control, but additionally indicates rigid and non-adaptive behavior of the whole population as a system, even under conditions in which everybody is together and similar by the interest.

4.2. Phase transition if affiliative behavior

We see a different picture in Figure 2 - the affiliation of the system is close to 1 as the distribution is skewed towards big clusters, and we see a remarkably small number of small clusters. It was found that the compatibility model exhibits a stochastic phase transition in the distribution of cluster sizes as a function of the sociability. Such a transition leads to the situation where almost all members of the population or group are similar by some «interest» or trait, and there is some veto upon different small «groups by interests». It looks like agents become more uniform and controlled by the majority of other agents.

For low populations, this transition bears many of the features of a continuous phase transition. There is a gradual increase in the order parameter across the transition. For large populations, the transition becomes more abrupt. The affiliation shows a very sharp transition from a level near 0 below a critical value, to a level close to 1 above this critical value. Studying different populations and varying different values of sociability parameter, we were able to estimate the critical sociability as the function \( S_c = P^{0.6} \), where \( P \) is the population size (Fig.3).

Sociability of agents appeared to be the main factor determining the affiliative behavior, and such behavior appears to be a group effect, independent of individual characteristics of a subject. It means that the breadth, depth and possibilities for keeping contacts with other agents or groups of agents as allowed in the organization or its division define the development of different kinds of group behavior.
The Compatibility model is closely related formally to percolation models [6] and to random graph models [9]. The qualitative behavior of both random graph and percolation models is determined by a critical parameter, \( c \). The effect of \( c \) is expressed by the size of the largest connected component, that is, the largest set of vertices which are continuously linked by edges. For values of \( c \) below a so-called critical threshold, the maximal cluster size remains small. For random graphs, this is on the order of \( \log n \), where \( n \) is the total number of vertices. At the critical threshold, there is a significant jump in the maximal cluster size, to the order of \( n^{2/3} \) for random graphs. Beyond the critical threshold, the maximal cluster size approaches order \( n \). Percolation models show similar qualitative behavior.

The difference between the models lies mostly in the fact that percolation models possess an underlying geometry which random graph models lack. Both models differ in one significant respect from the Compatibility model in that the connections, once established, are fixed. This is in sharp contrast to the dynamic nature of the links in the Compatibility model. The links in this model are not permanent and may be broken according to changing local conditions. This results in a degree of nonstationarity since the dynamics possesses a degree of past dependency. Also each agent is allowed to sustain links up to a maximal value which could result in the system becoming frustrated. Thus we obtain qualitative effects which are identical to those predicted by random graph theory, although these effects do differ in their point of emergence, occurring at much larger values of \( c \) and showing some dependence upon population size. Presumably, these latter effects reflect the dynamic nature of the process.

There is some similarity between the effects of this model and self-organized criticality [3], as well as Kauffman's model [8], and a model of Huberman and Glance [7], who studied collective behavior, using game theory. A difference between our and former models is that they considered a population of identical elements or very low diversity of elements (for example, 2 strategies in Huberman and Glance work). In addition to that, our model dealt with resource optimization between elements during their connections, while other models do not.

The similarity with the results of other models suggests that this clustering effect is not the unique result of the particular dynamics underlying the system, but instead is a result of a universal process underlying the formation of graphical connections in any system possessing a sufficient degree of stochasticity in their creation. The particular dynamics may well alter the quantitative properties of this behavior, such as the dependencies of the critical parameter, and the value of the critical threshold. But it will not alter the essential qualitative features.

**Figure 2.** Cluster Distribution Functions for population 400. The x-axis represents size of clusters, the y-axis represents number of such clusters normalized against the mode.
Thus this group dynamic shows the situation of separated small groups hardly having something similar between each other and more easily demonstrating some ignorance toward the main population (for example, in the case of physically separated individuals, or with the low density of population, where we have small sociability) and the effects of the emergence of global system behavior, effects of totalitarian control or the dominance of some idea, image, information over the other internal possibilities of the population (that we see in the case of someone's popularity, monopolies in economics, propaganda in mass media that reaches almost every citizen, or rules inside the organization, etc.)

We may be able to calculate the critical value of possible contacts that is necessary in order to maintain centralized management, in order to make people likely to accept some opinion and to establish a sense of psychological unity or group identity. Achieving and maintaining such a critical value of sociability in some group or society could help to prevent tension inside this community or to reduce the effect of some negative tendency. Historical contingencies play a major role in shaping the form that a group, organization or national identity will take, such as in the adoption of a socialist, communist, capitalist or fascist politics, but the basic underlying process responsible for the capacity for such global polities to arise is a universal of group dynamics. Thus our tendency to acquire such a social identity is not just a manifestation of the influence of our personality upon the group but also is a reflection of the influence of the group upon our personality.

The tendency towards the adoption of totalitarian political structures, corruption and monopolism even in the so-called enlightened Western democracies, maybe an expression of this deep group dynamic. That is why probably the best people and technologies of the world have big troubles in the fight against corruption and monopolism, especially now, when it appears on a trans-national level. On the other hand, the same unification tendencies, supported by computer technologies and access to the Internet (as an opportunity to significantly increase the number of contacts and an exchange of resources) gave speed and power to the development of science, humanitarian movements and market operations.

The situation in which the individuals within a society show little sociability leads to the formation of small groups, and effective isolation for a majority of the populace. In such a case, the society will express the rich diversity of traits, attitudes and beliefs, but will lack any sense of a coherent identity. With the trend towards so-called 'cocooning', following the introduction of the personal computer, separated rooms for kids, and along-standing individual houses, we have begun to witness a decline in socialization, particularly in the West. Is it possible that the gradual emergence of radical dissident splinter groups within the Western democracies is a manifestation, not of deep political schisms, but of our excessive emphasis
upon the individual, leading to diminished socialization? Is the adoption of such extreme ideologies perhaps the result of diminished socialization, rather than the converse?

4.3. Small town effect and optimal size of a group

The third interesting fact arising from the Compatibility model is the so-called «small town» phenomenon, which was found in the behavior of the small populations (up to 20 agents). Unlike the middle size (100-500) or large (more than 1000) populations, having structural stability of its dynamics, but very volatile structure of real connections, small populations demonstrated structural stability and very rigid structures of connections. As we looked at the behavior of six different «age» of clusters, usually we could observe that clusters do not live for a long time with the same composition, so most of our pictures show the life of the youngest group of clusters. However, in the small populations we could see that clusters of different age chose the same totalitarian strategy: even old clusters tended to keep the size, similar to the most unstable clusters (Fig.4).

We called it «the effect of small town»: there are no big possibilities to change the structure of connections in small isolated communities, so once established customs and groups tend to dominate for a long time over the individual characteristics of members of these communities. Stability of affiliation is also connected with the control of diversity «of interests», when an element, very different from the rest of the community will be forced to follow the characteristics of the big (often single) clusters. Psychologically we could easily understand such an effect of «country morality», but it was interesting to receive it mathematically.

The other phenomenon that we received in small populations is an effect of optimal group size, known in social psychology: agents tend to organize into groups with a size approximately of 7 members. Such an effect we observed personally in management training. We asked a group of 20 (sometimes 40 or even 100) people to carry out some logical task under the condition that the group come to a full consensus: the task will be solved only after the declaration of a correct solution and only if everyone from the group is in agreement with such a solution. It was funny to see that even under the demand for a global consensus, people created spontaneous groups of discussion, with an optimal size of 5-7 members, and many individuals shared one or another group, constantly changing their affiliation.

4.4. Diversity phenomena
A study of diversity as a parameter in various contexts has produced the following preliminary results.

1. The adaptation model allowed agents to change their «individuality» towards similarity through contact with other agents: agents received more resource for this similarity, paying only a little charge for this adaptation. The system was allowed to evolve to stationarity, as every agent could change its behavior. For behavior that was closer to the others, an agent received additional resource, and this, to our mind should lead to complete conformity for all populations. It was conjectured that the system should evolve towards a fixed point attractor, i.e. a homogeneity of the population, in which all agents have minimized their costs to the same degree. Surprisingly, it appeared that small regions or cluster of cells would persist in which these attributes would continue to fluctuate in a chaotic or possibly periodic manner. In other words, the system was frustrated, indicating symmetry breaking. Thus, even for such a simple model, individual differences persist even in the face of similarity among the majority of the population. This raises a deep question as to whether individual differences are a dynamical necessity.

2. The FD-1 model shows how inhomogeneity in the flow results in a spreading of properties within selected parameters, resulting in a polarization of abilities within the population of elements. For example, agents with the ability to accept significant resource (big income) and spend it (big outcome) while realizing many contacts with other agents (big sociability) more often play the function of conductor (in human life they could be journalists, teachers, clerks, postal workers, cashiers, salespeople). A big output of resources (big outcome) and small input (small income) characterize the producing and disposing sets of functions, while a big income and small outcome lead to the «condenser» set of functions (leader, selector, «warehouser», inhibitor, etc.).

Such a resource description of the roles that people could play in a group or organizations works not with the absolute value of sociability or resource flow values that an agent has, but with the ranking that each has inside the population. For example, the real value for an agent of some parameter could be average in comparison with other populations, but in a case in which almost no other agent has a larger value within this concrete population, this agent will function according to the type with high value of this parameter.

3. As was mentioned before, the FD-1 model, having the diversity of agent's limits on incoming and outcoming resource showed that strategies of spending resource play a bigger role in functional differentiation, than the amount of incoming resource. Elements in natural systems usually have access to a resource with the same probability, but spend it with various strategies and distribution.

![Figure 5. An example of the distribution of types in FD-2 model with a population 200 agents.](image-url)
4. The FD-2 model, studying the impact of such a characteristic of resource flow as a threshold of expenses, found that there is a ratio between elements with various thresholds, who survived after 5000 steps.

Elements with the low threshold spent more resource than elements with high threshold. We counted the percent of elements with threshold more than 50% of the maximum value (as receivers, high threshold elements) and less than 50% (as spenders, low threshold elements).

The distribution of functional types stayed within the limits: 20-25% of elements with low threshold and 75-80% of elements with high threshold. (Fig.5). This ratio between spenders and receivers in a population holds for various cases, but as soon as the sociability of agents is higher than the critical value, received in Compatibility model ($S_c = P^{6\sigma}$), the diversity of thresholds is close to the number of elements in population. Such diversity means basically that all agents are unique in their value of the threshold. The decrease of the diversity of threshold («reduced case») has 20-30 types of agents /values of threshold only) and the decease of sociability have similar effects: it leads to a decrease in the number of agents with the low threshold (spenders die out) and an increase in the number of receivers (elements which can hold a resource better survive more than others).

4.5. Relations between diversity, sociability and a size of a population.

Observation of relations between the parameters of diversity, sociability and the size of a population gave us the following results: the number 1 parameter is the size of the population, which partially determines the behavior of other parameters, but of course, needs certain values of these other parameters in order to produce an emergence of various phenomena.

The second «player» is sociability, that determines an apparent first order phase transition in affiliative behavior, which we discussed above. It appears as though the transition becomes sharper as the population size increases. We have evidence for the existence of a high-level group dynamics which is relatively independent of the low-level group dynamics, that is, the behavior of the individuals, and yet which influences such individual behavior. Group dynamics has long been viewed as an expression of the behavior of individuals, averaged over the collective. Here we see that groups possess their own dynamic, independent of the individual. If the individuals making up a large group or population are sufficiently sociable, then they will establish a network of connections within the group which effectively link every individual member to every other. This cohesiveness arises regardless of the degree of similarity among the individuals making up the group, and, as our preliminary evidence shows, paradoxically the greater the diversity among the members, the more readily this coherence is established.

A study of a limit on the general number of connections over all populations, which was done in the Resource model demonstrated that if the number of connections, totalled over the system is limited, but sufficient to integrate the system, it nevertheless does not integrate, even maximizing individual profit to form connections. Emergence of a system does not like the competition of the agents for the connections in every-step interaction. The speed of adaptation of elements to each other (their unification) is slower in the case with a limit on the total number of connections. A population with such a limitation tends to lose its total resource and does not form groups of connected elements. With such limits, agents cannot come to consensus and affiliate with other agents effectively. With the limit (sociability) on a number of individual connections the distance between elements stabilizes after the normal decrease and occupies a certain value. The system does not lose its resources and lives with group formations. It means that models with ordered or established connections have fewer chances to receive an emergence of living systems phenomena.

The other impact of sociability was demonstrated in the FD-2 model. It seems that sociability determines the asymptotic dynamics and variability of clusters and connections. Increase of sociability is connected with the increase of fluctuations and variability of clusters. Decrease of sociability leads to stationary modes.
The effect of diversity was weaker than sociability and population size but produced a second-order phase transition. In particular, there is a tendency for the transition to take place at lower maximal connections and for high diversity. In small populations the distribution function in the Compatibility model demonstrated strong peaking, most often towards large clusters but occasionally towards mid-range clusters (Figure 4). This behavior was not a simple function of population size. An increase in diversity brought about a return to the expected distribution.

In the FD-2 model the decrease of diversity («reduced case») led to a stronger dependency of distribution of functional types upon sociability. It corresponded to the Compatibility model, where a higher diversity of the population led to smaller clusters, and a smaller diversity was connected with the emergence of larger clusters.

In conclusion, we have just left to say that flexibility and resource-games of EVS might discover in the future much more interesting phenomena.

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References